These notes are intended to give a summary of relevant concepts from the lectures which are helpful to complete the tutorial sheet. It is not intended to cover the lectures thoroughly. Learning this content is not a replacement for working through the lecture material and the tutorial sheet.

**Topological ordering** \((x_1, \ldots, x_d)\) — For all \(x_i, x_j\) connected by a directed edge \(x_i \rightarrow x_j\), \(x_i\) should appear before \(x_j\) in the ordering.

**Ordered Markov property** — This is satisfied if \(\forall x_i \exists \pi_i\) s.t. \(x_i \perp \perp \text{pre}_i \setminus \pi_i | \pi_i\) where,

- \(\text{pre}_i\) is the set of nodes before \(x_i\) in a topological ordering
- \(\pi_i\) is a minimal subset of \(\text{pre}_i\)

For example, in graphs \(\pi_i = \text{parents}_i\)

**DAG connections**

<table>
<thead>
<tr>
<th>Connection</th>
<th>Serial</th>
<th>Diverging</th>
<th>Converging</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graph</td>
<td>(x \rightarrow z \rightarrow y)</td>
<td>(x \leftarrow z \rightarrow y)</td>
<td>(x \rightarrow z \leftarrow y)</td>
</tr>
<tr>
<td>(p(x, y))</td>
<td>(x \not\perp \perp y) - trail active</td>
<td>(x \not\perp \perp y) - trail active</td>
<td>(x \perp \perp y) - trail blocked</td>
</tr>
<tr>
<td>(p(x, y</td>
<td>z))</td>
<td>(x \perp \perp y</td>
<td>z) - trail blocked</td>
</tr>
</tbody>
</table>

**D-separation** — \(X \perp \perp Y | Z\) if every trail from \(\forall x \in X\) to \(\forall y \in Y\) is blocked by \(Z\)

Note, d-separation is not complete – it may not capture all independencies.

**Global directed Markov property** — All independencies by d-separation.

**Local directed Markov property** — \(x_i \perp \perp \text{nondesc}(x_i) \setminus \text{parents}(x_i) | \text{parents}(x_i)\)

**Markov blanket** \(\text{MB}(x_i)\) — The minimal set of variables \(\text{MB}(x_i)\) that makes \(x_i\) independent from all other variables.

\[
x_i \perp \perp X \setminus \{x_i \cup \text{MB}(x_i)\} | \text{MB}(x_i)
\]

\[
\text{MB}(x_i) = \text{parents}(x_i) \cup \text{children}(x_i) \cup \{\text{parents}(\text{children}(x_i)) \setminus x_i\}
\]